Section 4.3

4.6 Determine the frequency-domain function, $F(s)$, for each of the following time-domain functions, $f(t)$:

(a) $f(t) = 7.8$
(b) $f(t) = 3.2\cos 1000t$
(c) $f(t) = 120\sin 25t$
(d) $f(t) = 18t$
(e) $f(t) = 16e^{-8t}$
(f) $f(t) = 9e^{-3t}\sin 100t$
(g) $f(t) = 8.2te^{-2.5t}$
(h) $f(t) = 5e^{-7t}\cos 50t$
(i) $f(t) = 45e^{-5(t-6)}$
(j) $f(t) = 2\sin(t - 6)$
(k) $f(t) = 4.8e^{-5t}\cos(400t - 36^\circ)$
(l) $f(t) = 8\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt}$,

where $\dfrac{dx(0)}{dt} = 8,\ x(0) = -4$

(m) $f(t) = 12\int x\,dt + 17x$

4.7 Determine the time-domain function, $f(t)$, for each of the following frequency-domain functions, $F(s)$:

(a) $F(s) = \dfrac{6.7}{s^2}$

(b) $F(s) = \dfrac{25\omega}{s^2 + \omega^2}$

(c) $F(s) = \dfrac{45}{s + 72}$

(d) $F(s) = 345/s$

(e) $F(s) = \dfrac{650}{(s + 8)^2}$

(f) $F(s) = \dfrac{250\omega}{(s + 4)^2 + \omega^2}$

(g) $F(s) = \dfrac{82}{s(5s + 1)}$

(h) $F(s) = \dfrac{16(s + 5)}{(s + 5)^2 + \omega^2}$

(i) $F(s) = \dfrac{28s}{s^2 + \omega^2}$

(j) $F(s) = \dfrac{64\angle 48^\circ}{s + 8 - j16} + \dfrac{64\angle -48^\circ}{s + 8 + j16}$

Section 4.4

4.8 Complete the partial fraction expansion and find the inverse Laplace transformation of each of the following functions:

(a) $\dfrac{4(s+5)(s+7)}{s(s+3)(s+6)}$ (b) $\dfrac{2(s+5)}{(s+1)^2}$

(c) $\dfrac{s+2}{s^2+2s+4}$

4.10 Determine the transfer function, $I(s)/\theta(s)$, for the temperature transmitter described by the following differential equation:

$$8.6\frac{di}{dt} + i = 0.1\theta$$

where i = output current signal, mA
θ = input temperature signal, °C

4.12 Determine the transfer function, $\theta(s)/X(s)$, for a tubular heat exchanger similar to the one shown in Figure 2.3 and described by the following differential equation.

$$25\frac{d^2\theta}{dt^2} + 26\frac{d\theta}{dt} + \theta = 125x$$

where x = valve position, in.
θ = temperature of the fluid leaving the heat exchanger, °C

4.13 A manufacturing plant uses a liquid surge tank to feed a positive-displacement pump. The pump supplies a constant flow rate of liquid to a continuous heat exchanger. Determine the transfer function, $H(s)/Q(s)$, if the surge tank is described by the following equation:

$$h(t) = 0.5\int q(t)\,dt$$

where $h(t)$ = level of liquid in the surge tank, m
$q(t)$ = difference between the input flow rate and the output flow rate, m^3
t = time, s

4.14 The spring-mass-damping system shown in Figure 3.8 is described by the following differential equation:

$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + Kx = f$$

where m = mass, kg
R = dashpot resistance, N/(m/s)
K = spring constant, N/m
x = position of the mass, m
f = external force applied to the mass, N

Determine the transfer function, $X(s)/F(s)$, if

$$m = 3.2 \text{ kg}$$
$$R = 2.0 \text{ N/(m/s)}$$
$$K = 800 \text{ N/m}$$

4.16 An armature-controlled dc motor is sometimes used in speed and position control systems (see Figures 2.6 and 2.15). The dc motor operation is described by the following equations:

$$e = Ri + L\frac{di}{dt} + K_e\omega$$

$$i = \frac{q}{K_t}$$

$$q = J\frac{d\omega}{dt} + b\omega$$

where e = armature voltage, V
i = armature current, A
ω = motor speed, rad/s
q = motor torque, N · m
J = moment of inertia of the load, kg · m²
b = damping resistance of the load, N · m/ (rad/s)
R = armature resistance, Ω
L = armature inductance, H
K_e = back emf constant of the motor, V/(rad/s)
K_t = torque constant of the motor, N · m/A

A small permanent-magnet dc motor has the following parameter values:

$$J = 8 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$b = 3 \times 10^{-4}\ \mathrm{N \cdot m/(rad/s)}$$
$$R = 1.2\ \Omega$$
$$L = 0.020\ \mathrm{H}$$
$$K_e = 5 \times 10^{-2}\ \mathrm{V/(rad/s)}$$
$$K_t = 0.043\ \mathrm{N \cdot m/A}$$

Substitute these parameters into the preceding equations to obtain the exact differential equations of the dc motor. Determine the transfer function, $\Omega(s)/E(s)$, by transforming all three equations into frequency-domain algebraic equations. Use algebraic operations to obtain the ratio of Ω/E, which is the desired transfer function.

Homework 4.1

Find the inverse Laplace transform of the following.

1. $F(s) = \frac{3}{s^2+4}$
2. $F(s) = \frac{4}{(s-1)^3}$
3. $F(s) = \frac{2}{s^2+3s-4}$
4. $F(s) = \frac{3s}{s^2-s-6}$
5. $F(s) = \frac{2s+2}{s^2+2s+5}$
6. $F(s) = \frac{2s-3}{s^2-4}$
7. $F(s) = \frac{2s+1}{s^2-2s+2}$
8. $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$
9. $F(s) = \frac{1-2s}{s^2+4s+5}$
10. $F(s) = \frac{2s-3}{s^2+2s+10}$

Use the Laplace transform to solve the following ODEs

11. $y'' - y' - 6y = 0; \quad y(0) = 1,\ y'(0) = -1$
12. $y'' - 2y' + 2y = 0; \quad y(0) = 0,\ y'(0) = 1$
13. $y'' + 2y' + 5y = 0; \quad y(0) = 2,\ y'(0) = -1$
14. $y^{\mathrm{IV}} - y' = 0; \quad y(0) = 1,\ y'(0) = 1,\ y''(0) = 1,\ y'''(0) = 0$
15. $y'' - 2y' + 2y = \cos t; \quad y(0) = 1,\ y'(0) = 0$

Answers

1. $f(t) = \frac{3}{2}\sin 2t$
2. $f(t) = 2t^2e^t$
3. $f(t) = \frac{2}{5}e^t - \frac{2}{5}e^{-4t}$
4. $f(t) = \frac{9}{5}e^{3t} - \frac{6}{5}e^{-2t}$
5. $f(t) = 2e^{-t}\cos 2t$
6. $f(t) =??$
7. $f(t) = 2e^t\cos t + 3e^t\sin t$
8. $f(t) = 3 + 5\cos 2t - 2\sin 2t$
9. $f(t) = -2e^{-2t}\cos t + 5e^{-2t}\sin t$
10. $f(t) = 2e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t$
11. $y(t) = \frac{1}{5}(e^{3t} + 4e^{-2t})$
12. $y(t) = e^t\sin t$
13. $y(t) = 2e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t$
14. $y(t) = \frac{e^t+e^{-t}}{2}$
15. $y(t) = \frac{1}{5}(\cos t - 2\sin t + 4e^t\cos t - 2e^t\sin t)$

3.7 Find the time function corresponding to each of the following Laplace transforms using partial-fraction expansions:

(a) $F(s) = \frac{2}{s(s+2)}$

(b) $F(s) = \frac{10}{s(s+1)(s+10)}$

(c) $F(s) = \frac{3s+2}{s^2+4s+20}$

(d) $F(s) = \frac{3s^2+9s+12}{(s+2)(s^2+5s+11)}$

(e) $F(s) = \frac{1}{s^2+4}$

(f) $F(s) = \frac{2(s+2)}{(s+1)(s^2+4)}$

(g) $F(s) = \frac{s+1}{s^2}$

(h) $F(s) = \frac{1}{s^6}$

(i) $F(s) = \frac{4}{s^4+4}$

(j) $F(s) = \frac{e^{-s}}{s^2}$

3.8 Find the time function corresponding to each of the following Laplace transforms:

(a) $F(s) = \frac{1}{s(s+2)^2}$

(b) $F(s) = \frac{2s^2+s+1}{s^3-1}$

(c) $F(s) = \frac{2(s^2+s+1)}{s(s+1)^2}$

(d) $F(s) = \frac{s^3+2s+4}{s^4-16}$

(e) $F(s) = \frac{2(s+2)(s+5)^2}{(s+1)(s^2+4)^2}$

(f) $F(s) = \frac{(s^2-1)}{(s^2+1)^2}$

(g) $F(s) = \tan^{-1}(\frac{1}{s})$

3.9 Solve the following ODE using Laplace transforms:

(a) $\dddot{y}(t) + \dot{y}(t) + 3y(t) = 0; y(0) = 1, \dot{y}(0) = 2$

(b) $\ddot{y}(t) - 2\dot{y}(t) + 4y(t) = 0; y(0) = 1,\ \dot{y}(0) = 2$

(c) $\ddot{y}(t) + \dot{y}(t) = \sin t; y(0) = 1,\ \dot{y}(0) = 2$

(d) $\ddot{y}(t) + 3y(t) = \sin t; y(0) = 1,\ \dot{y}(0) = 2$

(e) $\ddot{y}(t) + 2\dot{y}(t) = e^t; y(0) = 1,\ \dot{y}(0) = 2$

(f) $\dddot{y}(t) + y(t) = t; y(0) = 1,\ \dot{y}(0) = -1$

In Problems 5–21 through 5–30, determine the inverse Laplace transforms of the functions given using Table 5–1 (and Table 5–2, if appropriate).

5–21 $V(s) = \dfrac{8}{s+3}$

5–22 $I(s) = 2 + \dfrac{3}{s} + \dfrac{4}{s^2}$

5–23 $F(s) = \dfrac{32}{s^2 + 64}$

5–24 $I(s) = \dfrac{10s}{s^2 + 25}$

5–25 $V(s) = \dfrac{6s + 15}{s^2 + 9}$

5–26 $I(s) = \dfrac{8 - 3s}{s^2 + 4}$

5–27 $F(s) = \dfrac{4s - 22}{s^2 + 4s + 29}$

5–28 $V(s) = \dfrac{100s}{s^2 + 200s + 50{,}000}$

5–29 $V(s) = \dfrac{s + 2}{s + 1}$

5–30 $F(s) = \dfrac{\pi}{s^2 + \pi^2}(1 + \varepsilon^{-s} + \varepsilon^{-2s} + \varepsilon^{-3s} + \cdots)$ (Sketch the time function.)

In Problems 5–31 through 5–48, determine the inverse transforms of the functions given using partial fraction expansion and the related procedures of Sections 5–6 through 5–8.

5–31 $F(s) = \dfrac{9s + 23}{(s + 2)(s + 3)}$

5–32 $I(s) = \dfrac{7s + 23}{(s + 3)(s + 4)}$

5–33 $V(s) = \dfrac{3s^2 + 13s + 8}{s(s^2 + 3s + 2)}$

5–34 $F(s) = \dfrac{4s^2 + 6s + 3}{s(s^2 + 4s + 3)}$

5–35 $V(s) = \dfrac{20s + 56}{4s^2 + 24s + 32}$

5–36 $F(s) = \dfrac{50s + 15{,}000}{s^2 + 300s + 2 \times 10^4}$

5–37 $I(s) = \dfrac{2(s + 2)}{(s + 1)(s^2 + 4s + 13)}$

5–38 $F(s) = \dfrac{50s + 100}{(s + 1)(2s^2 + 8s + 26)}$

5–39 $V(s) = \dfrac{120(s + 1)}{(4s + 8)(2s^2 + 50)}$

5–40 $I(s) = \dfrac{10}{(s^2 + 4)(s^2 + 2s + 5)}$

5–41 $V(s) = \dfrac{20(s^2 + 4)}{s(s + 1)(s^2 + 2s + 2)}$

5–42 $V(s) = \dfrac{100s^2}{(s^2 + 3s + 2)(s^2 + 2s + 2)}$

5–43 $I(s) = \dfrac{2s + 5}{(s^2 + 3s + 2)(s^2 + 6s + 25)}$

5–44 $I(s) = \dfrac{20s + 10}{s^4 + 5s^2 + 4}$