Solve the following differential equations using Laplace transforms. You will need to use MATLAB to solve these equations:

1) \[ \frac{d^2}{dt^2} x(t) + 3 \frac{d}{dt} x(t) + 2x(t) = u(t), \quad x(0) = 3, \ x'(0) = 0 \]

2) \[ 5 \frac{d^2}{dt^2} x(t) + 3 \frac{d}{dt} x(t) - x(t) = \sin t \ u(t), \quad x(0) = 0, \ x'(0) = 2 \]

3) \[ \frac{d^3}{dt^3} x(t) + 10 \frac{d^2}{dt^2} x(t) + 15x(t) = t^2 u(t), \quad x(0) = 10, \ x'(0) = 1, \ x''(0) = -5 \]

Each solution should contain the following information:

a. A derivation of \( X(s) = \frac{B(s)}{A(s)} \)

b. A list of the zeros and poles of \( X(s) \)

c. A rough sketch of the location of the zeros and poles in the complex plane (indicate zeros by the symbol “O” and poles by the symbol “X”)

d. Circle the dominant pole and indicate whether the system is stable or unstable

e. A partial fraction expansion \( X(s) = c_1/(s - p_1) + c_2/(s - p_2) + \ldots \)

f. The solution \( x(t) = c_1 \exp(p_1 t) + c_2 \exp(p_2 t) + \ldots \)

Problem 4 is on the next page.
4) The Laplace transform $X(s) = B(s)/A(s)$ produces the following pole-zero diagram. Poles are represented by the X’s, zeroes are represented by the O’s:

![Pole-Zero Diagram for Problem 4](image)

a. Reconstruct the Laplace transform $X(s) = B(s)/A(s)$ from the poles and zeroes shown above. Use MATLAB to express $A(s)$ and $B(s)$ as polynomials of $s$.

b. Identify the dominant pole and determine whether the system response will be stable or unstable.

c. Use MATLAB to find the solution for $x(t)$. 