General analysis of $v_o(t)$ in terms of switching functions.

HB \quad v_o(t) = \frac{V_S}{2} (2h_1 - 1)

FB \quad v_o(t) = V_S (h_1 - h_2)

Bipolar vs. Unipolar switching

Bipolar Switching: let \quad h_2 = 1 - h_1(\omega t)

Unipolar Switching: let \quad h_2 = h_1(\omega t - \pi) \quad \text{(Unipolar is only applicable to the FB.)}

Specific analysis of $v_o(t)$ in terms of switching functions if bipolar switching is used.

HB \quad v_o(t) = \frac{V_S}{2} (2h_1 - 1)

FB \quad v_o(t) = V_S (2h_1 - 1)

Specific analysis of $v_o(t)$ in terms of switching functions if unipolar switching is used.

FB \quad v_o(t) = V_S [h_1(\omega t) - h_1(\omega t - \pi)]

Because the shift angle $\pi$ is measured at the fundamental output frequency, this implies that all harmonic content at even multiples of the fundamental frequency is canceled in the output waveform, while the odd harmonic content remains the same as if bipolar switching had been used.
Figure 8-5  Pulse-width modulation.

\( f_1 = \) desired output frequency (see reference waveform)
\( f_S = \) switching frequency (see triangle carrier waveform) = \( m_f f_1 \)
\( m_a = \) modulation index (ratio of reference amplitude to triangle amplitude)
Choosing \( m_f \) for a single-phase inverter with bipolar switching (such as in Fig. 8-5)

1) Choose \( m_f \) as an odd integer \( \geq 9 \) (triangle carrier synchronized to reference waveform), or

2) Keep \( m_f \) quite large (\( > 21 \)) and synchronization is not necessary.

### Table 8-1 Generalized Harmonics of \( v_{ho} \) for a Large \( m_f \)

<table>
<thead>
<tr>
<th>( h )</th>
<th>( m_a )</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( m_f )</td>
<td>1.242</td>
<td>1.15</td>
<td>1.006</td>
<td>0.818</td>
<td>0.601</td>
</tr>
<tr>
<td>1</td>
<td>( m_f \pm 2 )</td>
<td>0.016</td>
<td>0.061</td>
<td>0.131</td>
<td>0.220</td>
<td>0.318</td>
</tr>
<tr>
<td>1</td>
<td>( m_f \pm 4 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.018</td>
</tr>
<tr>
<td>2</td>
<td>( m_f \pm 1 )</td>
<td>0.190</td>
<td>0.326</td>
<td>0.370</td>
<td>0.314</td>
<td>0.181</td>
</tr>
<tr>
<td>2</td>
<td>( m_f \pm 3 )</td>
<td>0.024</td>
<td>0.071</td>
<td>0.139</td>
<td>0.212</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( m_f \pm 5 )</td>
<td></td>
<td></td>
<td>0.013</td>
<td>0.033</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( m_f )</td>
<td>0.335</td>
<td>0.123</td>
<td>0.083</td>
<td>0.171</td>
<td>0.113</td>
</tr>
<tr>
<td>3</td>
<td>( m_f \pm 2 )</td>
<td>0.044</td>
<td>0.139</td>
<td>0.203</td>
<td>0.176</td>
<td>0.062</td>
</tr>
<tr>
<td>3</td>
<td>( m_f \pm 4 )</td>
<td>0.012</td>
<td>0.047</td>
<td>0.104</td>
<td>0.157</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( m_f \pm 6 )</td>
<td></td>
<td></td>
<td>0.016</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( m_f \pm 1 )</td>
<td>0.163</td>
<td>0.157</td>
<td>0.008</td>
<td>0.105</td>
<td>0.068</td>
</tr>
<tr>
<td>4</td>
<td>( m_f \pm 3 )</td>
<td>0.012</td>
<td>0.070</td>
<td>0.132</td>
<td>0.115</td>
<td>0.009</td>
</tr>
<tr>
<td>4</td>
<td>( m_f \pm 5 )</td>
<td></td>
<td></td>
<td>0.034</td>
<td>0.084</td>
<td>0.119</td>
</tr>
<tr>
<td>4</td>
<td>( m_f \pm 7 )</td>
<td></td>
<td></td>
<td></td>
<td>0.017</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Note: \( (\tilde{V}_{ho})_{0.5Vdc} = (\tilde{V}_{ho})_{0.5Vdc} \) is tabulated as a function of \( m_a \).

The table entries are normalized peak values of output voltage. The heading across the top is modulation index. The heading along the side is frequency, expressed as the harmonic number "h." The normalization base for the voltages is as follows:

\[
V_{BASE} = \frac{1}{2} V_{dc} \text{ for the half-bridge, and}
\]

\[
V_{BASE} = V_{dc} \text{ for the full-bridge.}
\]

These table entries are the harmonic content in the switching function \((2h_f - 1)\).
Figure 8-15  PWM with unipolar voltage switching (single phase).
Choosing $m_f$ for a single-phase inverter with unipolar switching (such as in Fig. 8-15)

1) Choose $m_f$ as an even integer $\geq 10$ (triangle carrier synchronized to reference waveform), or

2) Keep $m_f$ quite large ($> 22$) and synchronization is not necessary, and this will lead to

3) Harmonics in every other block of Table 8-1 will be deleted. The blocks deleted will be those for which the harmonic number is even.

<table>
<thead>
<tr>
<th>$m_f$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.242</td>
<td>1.15</td>
<td>1.000</td>
<td>0.818</td>
<td>0.609</td>
</tr>
<tr>
<td>$m_f \pm 2$</td>
<td>0.076</td>
<td>0.061</td>
<td>0.131</td>
<td>0.220</td>
<td>0.318</td>
</tr>
<tr>
<td>$m_f \pm 4$</td>
<td>0.048</td>
<td>delete</td>
<td>delete</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2m_f \pm 1$</td>
<td>0.190</td>
<td>0.326</td>
<td>0.370</td>
<td>0.314</td>
<td>0.181</td>
</tr>
<tr>
<td>$2m_f \pm 3$</td>
<td>0.024</td>
<td>0.071</td>
<td>0.139</td>
<td>0.212</td>
<td></td>
</tr>
<tr>
<td>$2m_f \pm 5$</td>
<td>0.013</td>
<td>0.033</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3m_f \pm 1$</td>
<td>0.335</td>
<td>0.123</td>
<td>0.083</td>
<td>0.071</td>
<td>0.013</td>
</tr>
<tr>
<td>$3m_f \pm 2$</td>
<td>0.044</td>
<td>0.139</td>
<td>0.203</td>
<td>0.176</td>
<td>0.062</td>
</tr>
<tr>
<td>$3m_f \pm 4$</td>
<td>0.012</td>
<td>0.047</td>
<td>0.104</td>
<td>0.157</td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 1$</td>
<td>0.163</td>
<td>0.157</td>
<td>0.008</td>
<td>0.105</td>
<td>0.068</td>
</tr>
<tr>
<td>$4m_f \pm 3$</td>
<td>0.012</td>
<td>0.070</td>
<td>0.132</td>
<td>0.115</td>
<td>0.009</td>
</tr>
<tr>
<td>$4m_f \pm 5$</td>
<td>0.034</td>
<td>0.084</td>
<td>0.119</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 7$</td>
<td>0.017</td>
<td>0.050</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: \( \frac{(V_{in})}{I/2} = \frac{(V_{in})}{\sqrt{2}V_d} \) is tabulated as a function of $m_f$.

The non deleted table entries are the harmonic contents in the switching function \([h_1 - h_1(e^{i\omega t - \pi})]\). These are given as normalized peak values. Unipolar is applicable only to the full-bridge inverter, so the normalization base will be the total dc side voltage.
Three Phase Inverter

1) Three switching functions are needed. Use one triangle carrier wave and a 3-phase set of reference sinusoids.

2) Pick $m_f$ to be an odd multiple of 3, and synchronize the carrier to the reference waveform if $m_f$ is low.

3) For large $m_f$, synchronization is not necessary.

There will be harmonic cancellations in the line-line voltages if these choices are made. A neutral can be created by splitting the dc source, but this should not normally be done.
Figure 8-22  Three-phase PWM waveforms and harmonic spectrum.
Table 8-2  Generalized Harmonics of $v_{LL}$ for a Large and Odd $m_f$ That Is a Multiple of 3.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m_a$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.122</td>
<td>0.245</td>
<td>0.367</td>
<td>0.490</td>
<td>0.612</td>
</tr>
<tr>
<td>$m_f \pm 2$</td>
<td></td>
<td>0.010</td>
<td>0.037</td>
<td>0.080</td>
<td>0.135</td>
<td>0.195</td>
</tr>
<tr>
<td>$m_f \pm 4$</td>
<td></td>
<td></td>
<td>0.005</td>
<td></td>
<td></td>
<td>0.011</td>
</tr>
<tr>
<td>$2m_f \pm 1$</td>
<td></td>
<td>0.116</td>
<td>0.200</td>
<td>0.227</td>
<td>0.192</td>
<td>0.111</td>
</tr>
<tr>
<td>$2m_f \pm 5$</td>
<td></td>
<td></td>
<td>0.008</td>
<td></td>
<td></td>
<td>0.020</td>
</tr>
<tr>
<td>$3m_f \pm 2$</td>
<td></td>
<td>0.027</td>
<td>0.085</td>
<td>0.124</td>
<td>0.108</td>
<td>0.038</td>
</tr>
<tr>
<td>$3m_f \pm 4$</td>
<td></td>
<td></td>
<td>0.007</td>
<td>0.029</td>
<td>0.064</td>
<td>0.096</td>
</tr>
<tr>
<td>$4m_f \pm 1$</td>
<td></td>
<td>0.100</td>
<td>0.096</td>
<td>0.005</td>
<td>0.064</td>
<td>0.042</td>
</tr>
<tr>
<td>$4m_f \pm 5$</td>
<td></td>
<td></td>
<td>0.021</td>
<td>0.051</td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 7$</td>
<td></td>
<td></td>
<td></td>
<td>0.010</td>
<td>0.030</td>
<td></td>
</tr>
</tbody>
</table>

Note: $(v_{LL})_h/V_d$ are tabulated as a function of $m_a$ where $(v_{LL})_h$ are the rms values of the harmonic voltages.

Table entries are normalized rms values of harmonic voltages in the line-line voltage set for the 3-phase FB inverter. Normalization base is dc-side voltage. Pick switching frequency as a large, odd multiple of three times the output frequency. ($m_f > 9$)

R. J. King
Nov. 20, 2012