MIME 3300 Analytical Linkage Synthesis (Design)

Objective: Given the desired output motion of a linkage, determine the linkage that produces this motion

Function generation: Correlation of an input motion with an output motion in a mechanism. Function generator is a black box that delivers some predictable output in response to a known input.
**Path generation:** Control of a *point* so that it follows a prescribed path

![Position of 4-bar linkage (open solution)](image)

**Motion generation:** Control of a *line* so that it assumes prescribed set of sequential positions

![Diagram of a 4-bar linkage](image)
Procedure for synthesis of a linkage

Specifications

Experience, judgment

Type synthesis: determine best type of linkage but do not worry about the dimensions

Algorithm and judgment

Dimensional synthesis: determine dimensions

Precision points (positions): Points (positions) prescribed for successive locations of the output

More points → more equations

Graphical synthesis: Use graphical procedures to determine linkage

Analytical synthesis: Use math
Design of slider-crank mechanism

Q = time ratio = time of working stroke/time of return stroke = $\alpha/\beta$

$\theta = \text{imbalance angle} = \pi \cdot (Q-1)/(Q+1)$

Problem definition; Given, stroke, Q, $r_2$, find: $r_3$, e

Solution

a. Find angles $\alpha$, $\beta$
b. Use eq. 

\[ r_3 = \sqrt{\frac{0.5 \text{stroke}^2 - r_2^2(1 + \cos((\alpha - \beta)/2))}{1 - \cos((\alpha - \beta)/2)}} \] (1)

c. Compute \( e \)

**Limiting case:** time of working stroke = time of return stroke

\( Q=1, \alpha = \beta \). Using cosine law for triangle in figure: \( r_2 = \text{stroke}/2 \). \( r_3 \) can assume any value greater or equal to \( r_2 \).

Example: \( r_2=7.4 \text{ mm}, Q=1.25, \text{stroke}=16 \text{ mm} \)

Solution \( r_3=19 \text{ mm}, e=6.6 \text{ mm} \)
Graphical solution

1) Compute $\alpha - \beta$
2) Guess a value of $r_3$
3) Draw circles centered about A and B with radii $r_3-r_2$, and $r_2+r_3$ respectively
4) Find the intersection of these circles
5) Check if $\epsilon = \alpha - \beta$. If yes stop otherwise change guess of $r_3$ and repeat steps 2-5 until the equation in 5 is satisfied.
Design of crank rocker mechanism:
Find a four-bar linkage to achieve desired throw angle, $\phi$, for the rocker and an imbalance angle $\theta$. 
Fully extended position
Folded position
Geometry and important equations

\[ O_2C_1 = b - a \]
\[ O_2C_2 = b + a \]
\[ O_2C_2 - O_2C_1 = 2a \]
where \( a \) = crank length
\( b \) = connecting rod length
\( c \) = rocker length
Steps:
1. Locate O₄
2. Select feasible c
3. Draw two positions of rocker separated by throw angle φ
4. Draw line M through C₂
5. Draw line N through C₁ at an angle equal to the imbalancel angle θ relative to M
6. Find O₂, intersection of M and N
7. Find a
8. Find b