Multiaxial stress effects on fatigue behavior of filled natural rubber

W.V. Mars a, A. Fatemi b,*

a Cooper Tire and Rubber Company, Findlay, OH 45840, USA
b Mechanical, Industrial and Manufacturing Engineering, The University of Toledo, Toledo, OH 43606-3390, USA

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Abstract

This work explores multiaxial stress effects on fatigue crack nucleation and growth in filled natural rubber based on experiments using short thin-walled cylindrical specimens subjected to axial and twist displacements. Cyclic stress–strain response exhibits significant initial softening relative to the monotonic response, followed by a more gradual additional softening. Irreversible breakage of various types of bonds is believed to cause the initial softening, while the presence of fillers and their influence on network chain breakage is believed to cause the additional softening. Crack nucleation planes and crack growth paths were monitored and cracks were observed to form on specific plane(s). Correlations of crack nucleation lives and growth rates with respect to the peak maximum principal strain were obtained and are discussed. © 2005 Published by Elsevier Ltd.

Keywords: Crack nucleation and growth; Filled natural rubber; Multiaxial stress states

1. Introduction

Rubber’s ability to withstand very large strains without permanent deformation makes it ideal for many applications such as tires, vibration isolators, seals, hoses, and belts. Since these applications impose large static and time-varying strains, durability and therefore mechanical fatigue is often the primary consideration.

Typically, the fatigue failure process involves a period during which cracks nucleate in regions that were initially free of observed cracks, followed by a period during which nucleated cracks grow to the point of failure. Analysis approaches that are currently available for predicting fatigue life in rubber, including both crack nucleation as well as crack growth approaches, are reviewed in Ref. [1]. The crack growth approach has been studied and used extensively. A major practical challenge in applying the crack growth approach to rubber is computation of the energy release rate associated with the crack of interest, and predicting the location and path of the fastest growing crack, especially when the geometry and loading are complicated. Robust numerical procedures are inevitably required, but are not widely available.

When the crack of interest is small, another problem is determining the initial location and size of a crack. Small flaws are often of particular importance, since most of a component’s life may be spent on the growth of small flaws. For uniaxial situations in which failure initiates from a small flaw, the strain energy density can be used to estimate the energy release rate of the flaw, from which fatigue life can be computed, given the fatigue crack growth curve. For multiaxial situations, the strain energy density is not generally appropriate because not all of the energy is available to be released by the growth of a flaw.

The nucleation approach has received less attention than the crack growth approach in the literature, although many engineers still use this approach for its simplicity and familiarity. The nucleation approach is advantageous for analyzing the spatial distribution of fatigue life, since it is based on quantities that are defined at a material point, in the sense of continuum mechanics. In rubber, uniaxial fatigue life results are most commonly correlated based on maximum principal strain (or stretch), and strain energy density. Neither of these parameters has been robustly successful in correlating results from different strain states, particularly simple tension and equibiaxial tension.

Many factors influence the fatigue behavior of rubber. A review of these factors is given in Ref. [2]. They include various aspects of the mechanical loading history, environmental effects (elevated temperatures and the presence of oxygen or ozone), effects of rubber formulation (elastomer type, filler type and volume fraction, antidegradants, curatives,
and vulcanization), and effects due to dissipative aspects of the constitutive response of rubber. While a large number of factors have been studied individually, the ability to integrate such diverse results into accurate fatigue life predictions remains an ambitious and elusive goal.

A primary consideration relating to the mechanical load history is that for fatigue failure to occur, a fluctuating load must be present. It is traditional, however, in the rubber literature, to present fatigue life and fatigue crack growth data against the maximum loading, which, by itself is ambiguous about the presence of a fluctuating load. Practical and theoretical reasons that this tradition has persisted are presented in Ref. [2]. Rubber’s fatigue behavior is extremely sensitive to both the maximum and minimum cyclic load limits.

For \( R = 0 \) cycles, the relationship between maximum load and fatigue crack growth rate (or fatigue life) exhibits a threshold, below which crack growth and fatigue failure do not occur. There is also a critical load above which unstable fracture occurs. Between these extremes, rubber exhibits a substantially power-law relationship between the load and the crack growth rate or fatigue life. Lake and Lindley identified four distinct regimes of fatigue crack growth behavior, based on the maximum energy release rate per cycle [3].

Other aspects of the mechanical load history including the effects of static loaded periods ("annealing"), load sequence, multiaxiality, frequency, and loading waveform are also discussed in Ref. [2]. The effects of variable amplitude loading have not been studied. In particular, there is a need to identify and apply a suitable cycle-counting algorithm capable of parsing complex loading histories into simpler histories that may be compared with fatigue data based on material testing.

While many uniaxial studies of rubber deformation and characterization exist, theories and experimental data of rubber subject to multiaxial stressing are less common, particularly for the case of cyclic loading. This paper discusses crack nucleation and growth in a filled natural rubber compound subjected to multiaxial loading. In applications, filled rubbers are most often used in mechanically severe applications due to their generally superior properties and lower cost. While unfilled rubbers exhibit nearly ideal non-linear elastic behavior, filled rubbers exhibit more complex history dependent behavior.

Uniaxial fatigue behavior of this rubber compound was investigated using fatigue macro-crack nucleation as well as fatigue crack growth experiments in Ref. [4]. Simple tension specimens were used for crack nucleation experiments and planar tension specimens were used for crack growth experiments. An advantage of the planar tension specimen is that under displacement control, the energy release rate is independent of crack size and the fatigue crack growth rate is constant [5,6]. Experimental macro-crack nucleation lives were correlated by using the applied peak (or range of) strain. Similar correlations are obtained by using peak (or range of) stress or strain energy density. Fatigue crack growth rates were satisfactorily correlated by using the range of the applied energy release rate as the correlating parameter.

Using an effective initial flaw size, good agreement was obtained between nucleation life properties obtained from simple tension specimens, and fatigue crack growth properties obtained from planar tension specimens [4]. Unintentional flaws that exist in the virgin material are a major source of fatigue life variability in macro-crack nucleation tests. Crack growth testing avoids this source of variability since the controlled/observed crack size is generally much larger than naturally occurring unintentional flaws.

Depending on polymer type and the presence of fillers, the effect of minimum or mean loading may be either beneficial or harmful. In strain crystallizing rubbers, increased minimum strain is beneficial. In non-strain-crystallizing rubbers, increased time-average strain can be harmful, due to steady crack growth. Indeed, in such rubbers, failure under a static load due to steady crack growth must be considered.

The effect of \( R \) ratio (minimum strain) on uniaxial fatigue behavior of filled natural rubber used in this study is discussed in Ref. [7], where it is shown that a small positive \( R \) ratio can have a significant effect on fatigue life and crack growth rate. Macro-crack nucleation tests with \( R > 0 \) cycles resulted in about an order of magnitude increase in the high cycle fatigue life and about a factor of two increase in the low cycle fatigue life, as compared with \( R = 0 \) tests. The effect of \( R > 0 \) cycles in crack growth tests was to decrease the fatigue crack growth rates, particularly at low strain range, where crack growth rate was reduced by more than an order of magnitude.

Based on characteristic features exhibited by data from strain crystallizing rubbers, a simple phenomenological model for the effect of \( R \) ratio on uniaxial fatigue behavior was proposed in Ref. [7]. First, when plotted as a function of maximum loading both crack growth and fatigue life data tend to exhibit power-law behavior at a given \( R \) ratio. Second, the curves for the various \( R \) ratios tend to converge at a single point that lies on an asymptote associated with the static load at which the rate of crack growth goes from 0 to \( +\infty \). The model was applied both for fatigue crack growth curves and for Wohler-style (S-N style) fatigue life curves. The ability of the model to represent data over a wide range of conditions was demonstrated via analyses of experimental results generated by the authors, as well as by Lindley [8] and Cadwell et al. [9].

2. Experimental program

A specimen was developed in which simultaneous axial and shear strains are produced via independently controlled axial and twist displacements of rigid mounting rings. The specimen is a short, hollow cylinder of rubber, bonded between two steel mounting rings. The extent and effects of strain and stress field non-uniformity in the test specimen are investigated in Ref. [10]. The specimen geometry is sufficiently simple that a closed form expression for the strain–displacement relationship was successfully developed.

FEA was used to investigate the stress–load relationships, and the strain energy density calculations for the specimen. A non-zero hoop stress arises due to the constraint associated with the short specimen gauge length, and the fact that
the specimen is bonded to rigid mounting rings. The hoop stress can be quite large. However, calculation of the hoop stress is often not necessary, since neither strain-based nor strain energy density-based fatigue life parameters are affected by the hoop stress. Details of the specimen geometry, and its design and analysis are provided in Ref. [10].

Axial–torsion experiments were run on a servo-hydraulic, axial–torsion test frame under displacement-twist control. The multiaxial loading path types investigated, and the associated path designations that are used throughout this paper, are shown in Fig. 1. These include axial loading, torsion loading, proportional axial–torsion loading, and non-proportional axial–torision loading. Although a wide range of strain states and history types were investigated, there remains a significant range of states for which no data were generated. A 15% load drop relative to response at cycle 128 was used as the failure criterion in displacement-twist control tests. Details of the experimental procedure are discussed in Ref. [11].

All axial testing (path A) was conducted under $R=0$ conditions. Torsion testing was conducted at both $R=0$ (path B) and $R=−1$ (path C) conditions. Due to finite strain effects, pure torsion tests do not result in a state of pure shear strain. Three other types of $R=−1$ cyclic torsion tests were run to check the effects of static axial loading (paths J, K, and M). Proportional testing, in which the axial and twist displacements trace a linear path of constant slope through the origin, was conducted primarily for $R=0$ conditions (path D). Two ratios of axial to twist displacement were employed, designated as paths D1 and D2. Several proportional tests were also run with $R=−1$ (path E). Two nominally proportional tests were conducted in load control, path N, to check the effects on fatigue life relative to displacement control.

Non-proportional testing, in which the axial and twist components differed by a phase angle $\phi$, was also conducted. Paths D, G, H, and I correspond to tension–torsion tests with phase angles of 0, 45, 90, and 180°, respectively. Path L corresponds to a compression–torsion test in which the peak compression is reached 180° out-of-phase with the peak torsion. Two non-proportional tests were conducted in load control, using path O, to check the effect on fatigue life relative to displacement control.

3. Results and discussion

3.1. Deformation behavior

Characterization and modeling of stress–strain response under monotonic and cyclic multiaxial load histories for the filled natural rubber investigated are discussed in Ref. [11]. The cyclic stress–strain response differed from the monotonic response in several significant ways. Typical results for proportional path I tests are shown in Fig. 2, where the evolution of the axial and shear stress amplitudes with cycles are shown, as well as typical stress–strain curves at $N=128$. For comparison, experimentally obtained monotonic stress–strain curves are superimposed with the $N=128$ stress–strain curves.

The cyclic stress–strain response exhibits significant softening relative to the monotonic response of the virgin material. The magnitude of the softening depends on the maximum strain experienced. The larger the maximum strain experienced, the softer the stabilized response. The effect is known as the Mullins effect. Hyperelastic models do not capture this effect, but recent models capable of addressing it have been developed [12].

A large portion of the initial softening is accomplished within the first 10 cycles. This large initial decrease in material stiffness is believed to be associated with the irreversible breakage of various types of bonds in the elastomer-filler composite. After this short period of significant softening, the stress–strain response under constant-amplitude cyclic loading exhibits a more gradual logarithmic trend of additional softening. It is believed that this effect is associated with the presence of fillers in the rubber, and their influence on network chain breakage. In the final portion of the test, the rate of stiffness degradation increases rapidly, until the specimen is completely broken. The strong dependence of the softening on the presence of fillers is related to their strain-amplifying effects and is associated with damage, as demonstrated by McKenna and Zapas [13] and by Derham and Thomas [14]. It has also been suggested that Mullins effect damage translates into accelerated creep and a larger strain at failure [13].

The phase angle has an effect on the shape and area of the hysteresis loop, with 90° out-of-phase histories resulting in

![Fig. 1. Axial–torsion loading path designations. $\delta$, axial displacement; $\theta$, twist; $P$, axial load; $T$, torque.](image-url)
maximum hysteresis. However, the overall effect of the phase angle on the elastic stress–strain response is small. In addition, the evolution of stress amplitude for out-of-phase paths does not differ significantly from the evolution of in-phase paths. The softening is observed to affect the shear stress in proportion to its effect on the axial stress. These results suggest that the steady-state cyclic constitutive response is reasonably reversible and that an assumption of non-linear elasticity may be justified as a first approximation, independent of the phase between multiaxial load components.

3.2. Fatigue crack nucleation and early growth

For the same level of twist amplitude, $R_0=0$ torsion tests (path B) gave shorter fatigue lives than $R_0=-1$ torsion tests (path C) by a factor of eight. Proportional fatigue lives were

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Fig. 2. Axial and shear engineering stress amplitude evolution with cycles and cyclically stabilized axial and shear stress–strain curves for path I. For comparison, the pure axial, pure torsion, and proportional monotonic stress–strain curves are superimposed.
intermediate between the pure axial and pure torsion results. The proportional load-controlled tests (path N) were found to agree reasonably with proportional tests run under displacement control, reflecting the relatively high degree of elasticity of the rubber. Relative to tests using path D, the observed fatigue lives for tests with path F were generally longer, indicating the importance of crack closure. Fatigue lives for out-of-phase loading (path I) were generally longer than those for path D by a factor of about five, over all life regimes. This is due to the fact that combined out-of-phase $R=0$ axial and shear strain cycles result in reduced amplitude cycles of the maximum principal strain.

The overall fit of data from paths A–O is shown against peak maximum principal strain in Fig. 3. The maximum principal strain parameter derives entirely from the applied axial and twist displacements, without regard to the resulting load or torque histories. The peak maximum principal strain correlates with life reasonably well, and somewhat better than the maximum principal strain amplitudes. This is particularly interesting in light of the fact that fatigue failure cannot occur without fluctuation of the load.

The material exhibits the Mullins effect, which depends primarily upon the peak deformation previously attained. The fatigue process also depends heavily on events whose occurrence can only be determined on an instantaneous basis (such as occurrence of strain crystallization). Peak and minimum values correspond to specific instantaneous strain states, while the amplitude alone is ambiguous regarding the state of the material. When a large range of $R$ ratios must be considered, however, correlation to peak or amplitude alone will be highly inadequate. In such cases, the effects of both peak and $R$ ratio must be considered.

One general observation that may be made regarding the formation of cracks on the axial–torsion specimen surfaces, independent of strain history type, is that the cracks formed on a specific plane, or in certain cases, on several specific planes. For proportional, $R=0$ histories, the observed cracking plane was oriented perpendicular to the direction of maximum tensile strain. For more complex histories, preferred nucleation planes were still observed, but their relationship to the principal strain directions was sometimes different.

In most cases, crack growth exhibited substantial self-similarity, from the earliest stages of observed initiation, to the final stages when the crack growth rate was strongly influenced by specimen boundaries. Scanning electron microscopy (SEM) showed that surface cavities in the expected size range (of order 0.1 mm) are present. Inspection of a failed specimen gave direct evidence that nucleation proceeds from pre-existing crack nucleation sites. Experimentally observed crack growth histories exhibit substantially similar evolution with applied cycles. At specimen failure, cracks typically reached a length in the range of 1–10 mm. This range primarily reflects variation of the initial effective lengths of the cracks.

Traditional approaches for multiaxial fatigue crack nucleation analysis in rubber such as the maximum principal strain and the strain energy density, and more recently the maximum principal stress [15], are based on criteria that make no reference to a specific material failure plane. It is always possible to construct a non-proportional multiaxial history that holds the scalar equivalence criterion value constant while simultaneously varying the individual components of the history [16]. Scalar equivalence criteria, therefore, predict infinite life under certain kinds of non-proportional cyclic loading which actually result in finite life.

Ideally, an equivalence principle ought to have the property that a unique value of the associated parameter corresponds to a unique value of the fatigue life, regardless of specimen geometry or type of multiaxial loading. Equivalence parameters traditionally associated with crack nucleation analysis for rubber have not been able to do this across a wide range of strain states [17–19].

In addition, the traditional multiaxial fatigue theories are not sufficiently consistent or complete and do not account for crack closure. Therefore, an analysis approach that makes specific
reference to the failure plane, such as the cracking energy density criterion described in Refs. [16,20,21] is better suited in crack nucleation life analyses of multiaxial strain histories. Amongst the commonly used scalar equivalence criteria, however, the maximum principal strain criterion is superior in terms of both correlation to life, and in terms of association with observed failure planes [20].

It should be noted that the practice of applying scalar equivalence criteria to the analysis of proportional multiaxial loading cycles in metals, through the notion of a stress or strain 'amplitude tensor', is not appropriate in the analysis of rubber. This is due to non-linearity associated with finite strains and near-incompressibility. Instead, it is recommended to compute the amplitude of the time history of the criterion of interest.

3.3. Macroscopic fatigue crack growth

Typical examples of crack initiation and growth from a single typical specimen are shown in Figs. 4 and 5 for path D. The history is presented as a sequence of photographic images (Fig. 4), and as crack length plotted against cycles using linear as well as logarithmic scales (Fig. 5). The photographic images give a sense of the crack density at failure, which reflects the combined influences of the initial flaw density, and the subsequent evolution of the flaw density through processes such as crack coalescence and shielding [22].

All experimentally observed crack growth histories exhibit substantially similar evolution with applied cycles, whether viewed on linear or logarithmic scales. Characteristically, very

![Fig. 4. Crack nucleation and growth for proportional loading path D (specimen 73).](image-url)
few cracks were visible in the photographs for $N/N_f < 1/3$. When viewed on linear scale, it is seen from Fig. 5 that a relatively large range of crack sizes is developed at failure. More details of crack nucleation and growth observations, as well as failure plane predictions, can be found in Ref. [22].

Of primary interest in fatigue analysis is whether the fatigue crack growth rate under complex multiaxial conditions can be predicted based on tests conducted under simple uniaxial conditions. The uniaxial fatigue crack growth behavior for the compound used in this study was investigated using the planar tension specimen geometry [4]. As indicated earlier, this geometry (also known as the pure shear specimen) has proven to be especially useful in fracture mechanics tests for rubber. This specimen is short and wide such that lateral contraction is prevented by the grips, while an axial strain is introduced in the direction of the short dimension.

Observing that energy density varies approximately with the square of the maximum principal strain $\epsilon_1$ and the crack length $a$, a pseudo-energy release rate can be computed as $T_r = C_1 \epsilon_1^2 a$, where $C$ (equal to 9 MPa here) may be interpreted as a material constant with the same units as strain energy density, and related to Young’s modulus associated with the monotonic response. This equation can also be interpreted as the energy release rate corresponding to a strain intensity factor, $\epsilon_1 \sqrt{a}$, which may be said to characterize the spatial variation of the strain field near the crack tip. Fig. 6 shows correlation of fatigue crack growth rates computed from the crack growth histories of the axial–torsion specimen subjected to load histories in Fig. 1. The correlation is based on the energy release rate computed from $T_r$, which is based on maximum principal strain.

As can be seen, maximum principal strain (and the associated pseudo-energy release rate) gives relatively good agreement between multiaxial path types, even though correlations with the planar tension results are not as good. Note that in displacement controlled tests, use of the maximum principal strain parameter does not require any knowledge of constitutive stress–strain behavior. Because of the Mullins effect, the Young’s modulus used in calculating material constant $C$ in $T_r$ does not possess a unique value, as assumed.
1. The cyclic stress–strain response exhibits significant softening relative to the monotonic response, where the magnitude of the softening depends on the maximum strain experienced. Significant initial softening is believed to be associated with the irreversible breakage of various types of bonds, followed by a more gradual additional softening, believed to be associated with the presence of fillers and their influence on network chain breakage.

2. Observations of fatigue crack nucleation indicated that the precursors to fatigue failure in rubber are flaws that exist in the virgin material. Fatigue crack nucleation and growth occurred on preferred failure planes. For the strain histories investigated, this was usually the plane transverse to the direction of maximum principal strain.

3. Fatigue lives for proportional axial–torsion tests were intermediate between those associated with axial and torsion tests. For given peak axial and shear strain levels, the fatigue life of out-of-phase axial–torsion tests increases with phase angle $\phi$, for $0^\circ < \phi < 180^\circ$, and $R=0$ conditions. Among several commonly used scalar equivalence criteria considered, the maximum principal strain criterion provided the best correlations of fatigue lives.

4. Traditional approaches for multiaxial fatigue crack nucleation analysis in rubber are based on criteria that make no reference to a specific material failure plane. An analysis approach that makes specific reference to the failure plane such as the cracking energy density criterion, is better suited in crack nucleation life analyses of multiaxial strain histories involving a wide range of stress/strain states and crack closure.

5. Very few cracks were visible during the first 1/3 portion of the fatigue lives, whereas a relatively large number of
cracks with varied sizes developed with additional cycles, up to failure. The large range of crack sizes is believed to primarily reflect variation of the initial effective lengths of the cracks.

6. Observed growth rates of nucleated cracks under multiaxial loading were compared with estimates of the crack driving force based on maximum principal strain. Maximum principal strain provided relatively good correlation of results from the axial–torsion specimen, but did not match results from the planar tension specimen as closely.

References