4.9 Given $\sigma_{xx} = 150$ MPa, $\sigma_{xy} = 65$ MPa, $\sigma_{yy} = \sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$; therefore $\sigma_{zz} = 0$ is a principal stress. As in Prob. 4.7, the other two principal stresses are

$$\sigma_1 = \frac{150}{2} + \sqrt{\left(\frac{150}{2}\right)^2 + 65^2} = 75 + 99.25 = 174.25 \text{ MPa}$$

$$\sigma_3 = 75 - 99.25 = -24.25 \text{ MPa}; \text{ also } \sigma_2 = \sigma_{zz} = 0.$$

$$\therefore \sigma_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}(174.25 + 24.25) = 99.25 \text{ MPa}$$

(a) The Tresca criterion requires

$$\sigma_{\max} = \frac{1}{2} Y \therefore SF = \frac{Y}{2 \sigma_{\max}} = \frac{450}{2(99.25)} = 2.267$$

(b) The von Mises criterion requires [See Eq. (4.21)]

$$\frac{\sigma_1 - \sigma_2}{2\sqrt{3}} + \frac{(\sigma_2 - \sigma_3)^2}{3} + \frac{(\sigma_3 - \sigma_1)^2}{3} = \frac{1}{2} Y^2$$

Therefore,

$$SF = \frac{12Y}{\left(\frac{\sigma_1 - \sigma_2}{2\sqrt{3}} + \frac{(\sigma_2 - \sigma_3)^2}{3} + \frac{(\sigma_3 - \sigma_1)^2}{3}\right)^{1/2}} = \frac{12(450)}{\left[\frac{174.25 - 24.25}{2\sqrt{3}}\right]^{1/2}}$$

$$SF = 2.399$$

4.10 With $\varepsilon_{zz} = 0$, we have $(\varepsilon_{xx} = \varepsilon_{yy} = 0)$. Also $\sigma_{xx} = 60$ MPa, $\sigma_{xy} = 240$ MPa, and $\sigma_{yy} = -80$ MPa. Since $\varepsilon_{xx} = \varepsilon_{yy} = 0$, $\sigma_{xx} = \sigma_{yy} = 0$. Therefore $\sigma_{zz}$ is a principal stress; $\sigma_{zz} = 0.29(60+240) = 87$ MPa. Also as in Prob. 4.7, the other two principal stresses are

$$\sigma_1 = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{yy}^2} = 150 \pm 120.42 = 270.42, 29.58$$

Ordering the stresses $\sigma_1 = 270.42$ MPa, $\sigma_2 = 87$ MPa, $\sigma_3 = 29.58$ MPa.

Now, $\sigma_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}(\sigma_1 - \sigma_3) = 120.42$ MPa. Therefore,

$$\sigma_{\max} = \frac{1}{2} Y \therefore SF = \frac{Y}{2 \sigma_{\max}} = \frac{490}{2(120.42)} = 2.035.$$